DETERMINING THE OXYGEN DIFFUSIVITY IN HIGH-TEMPERATURE COMBUSTION PRODUCTS FROM TEST DATA ON THE BURNING OF COAL PARTICLES

S. V. Bukhman and E. Nurekenov

UDC 533.15+662.62

It is shown in the article that, if data on both the combustion time and the temperature of particles are given, a combined processing of these data can lead to an elimination of the stoichiometric ratio and yield a formula for the oxygen diffusivity in high-temperature combustion products:

$$D_\infty = \frac{d}{0.375\,\rho_\infty \ln(1+x_\infty)(q_1-q_2)} \frac{L^{3/4}}{1+L^{3/4}} \left[Q_1+Q_2+\frac{\rho_p d}{4\tau}(q_1-2q_2) \right], \quad (1)$$

with ρ_p, τ, d denoting respectively the density, the combustion time, and the diameter of a particle; ρ denoting the gas density; x denoting the relative mass concentration of oxygen; $L = T_\infty/T_p$ denoting the ratio of stream temperature to particle temperature; Q_1, Q_2 denoting respectively the convective and the radiative thermal flux emanating from a particle; and q_1, q_2 denoting the heat of combustion which produces CO_2 and CO respectively. All quantities with the subscript ∞ pertain to the boundary layer outside a particle and are referred to the temperature of the oncoming stream.

Formula (1) has been derived from the heat balance and the material balance on the basis of diffusive combustion in a stationary medium, taking into account the Stefan flux as well as the nonisothermal conditions.

Formula (1) is used for evaluating the test data of various authors on the combustion of coal dust.

When only data on the combustion rate of coal particles are given, the effect of kinetics and the indeterminacy of the stoichiometric ratio are eliminated from the analysis by processing the data obtained for temperatures above 1300°K. It is assumed that combustion at the coal surface produces CO.

Furthermore, the effect of secondary reactions at high temperatures is excluded from the analysis by taking data at low oxygen concentrations ($O_2 < 3\%$) or at high values of the mass transfer coefficient (fine particles). Another advantage of data obtained at low concentrations is that they reflect combustion under almost nonisothermal conditions.

The results indicate that at high temperatures (up to 1670°K) the oxygen diffusivity in combustion products can be described by a power equation with the exponent n = 1.75.

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SOLUTION OF THE MIXING PROBLEM FOR PETROLEUM PRODUCTS WITH MORE PRECISE BOUNDARY CONDITIONS

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UDC 622.692.43:532.72

Recent industrial and laboratory experiments have shown that, during sequential pumping of various petroleum products through the same pipeline, a certain quantity of mixture (so-called primary mixture) is formed already in the inner routes in the head pumping station. The quantity of this mixture, depending on the complexity and the length of suction routes as well as on the hydrodynamic conditions in the suction lines of pumps, may be of the same order as the quantity of mixture formed in the main segments of some pipelines. This is an important factor in the subsequent mixing of petroleum products. The authors analyze a problem the solution of which will allow one to estimate the effect of primary mixture on the mixing rate of successively pumped products flowing in a pipeline and to determine the size of the full mixture at the endpoint of a pipeline.

It is assumed that at time t = 0 the primary mixture occupies a pipeline segment of length $2a$ from the entrance point and that the empirical relation $c_2(x, 0) = 1/2(1 + \sin(\pi x/2a))$ represents the concentration distribution of liquid displaced in the zone of primary mixture. In a system of coordinates with the origin 0 at the volume center of the primary mixture and with the x-axis coinciding with the pipeline axis, then the mixing process at time $t \gg 0$ can be described by the following systems of equations based on the theory of turbulent diffusion:

$$\frac{\partial c_i}{\partial t} = \frac{\partial}{\partial x}\left(K_i \frac{\partial c_i}{\partial x}\right),\ t \in [0, \infty),\ x \in [a, \infty) \text{ at } i = 1,\ x \in [-a, a],\ i = 2,\ x \in (-\infty, -a],\ i = 3; \tag{1}$$

$$c_1(x, 0) = 1;\quad c_2(x, 0) = \frac{1}{2}\left(1 + \sin\frac{\pi x}{2a}\right);\quad c_3(x, 0) = 0;$$

$$\frac{\partial c_i(\infty, t)}{\partial x} = 0;\ c_1(a, t) = c_2(a, t);\ c_2(-a, t) = c_3(-a, t); \tag{2}$$

$$\frac{\partial c_1(a, t)}{\partial x} = \frac{\partial c_2(a, t)}{\partial x};\quad \frac{\partial c_2(-a, t)}{\partial x} = \frac{\partial c_3(-a, t)}{\partial x};\quad \frac{\partial c_3(-\infty, t)}{\partial x} = 0.$$

At $t \gg 0$ it is legitimate to assume $K_i = K = \text{const}$ and the solution to the problem will be

$$c_1(x, t) = 1 - \frac{1}{4}\left(\operatorname{erf} u + \operatorname{erf} v\right) + \frac{1}{8}e^{-w^2}\left\{\left[e^{-i\varphi}\operatorname{erf}(v - iw) + e^{i\varphi}\operatorname{erf}(v + iw)\right] + \left[e^{-i\psi}\operatorname{erf}(u - iw) + e^{i\psi}\operatorname{erf}(u + iw)\right]\right\}; \tag{3}$$

$$c_2(x, t) = \frac{1}{2} + \frac{1}{4}\left[\operatorname{erf}(-u) - \operatorname{erf} v\right] + \frac{1}{2}e^{-w^2}\sin\frac{\pi x}{2a} - \frac{1}{8}e^{-w^2}\left\{\left[e^{-i\psi}\operatorname{erf}(-u - iw) + e^{i\psi}\operatorname{erf}(-u + iw)\right] - \left[e^{-i\varphi}\operatorname{erf}(v - iw) + e^{i\varphi}\operatorname{erf}(-v + iw)\right]\right\}; \tag{4}$$

$$c_3(x, t) = \frac{1}{4}\left[\operatorname{erf}(-u) + \operatorname{erf}(-v)\right] - \frac{1}{8}e^{-w^2}\left\{\left[e^{i\psi}\operatorname{erf}(-u - iw) + e^{-i\psi}\operatorname{erf}(-u + iw)\right] + \left[e^{i\varphi}\operatorname{erf}(-v - iw) + e^{-i\varphi}\operatorname{erf}(-v + iw)\right]\right\}. \tag{5}$$

Equations (3)-(5) describe the concentration distribution of liquid displaced in the product separation zone and, therefore, allow one to determine the size of other than standard mixtures as a function of time t and parameters $2a$, K.

NOTATION

$c_i(x, t)$ is the dimensionless concentration of liquid displaced from a pipeline at a given stream zone;

$u = (x - a)/2\sqrt{Kt}$;

$v = (x + a)/2\sqrt{Kt}$;

$w = \pi\sqrt{Kt}/2a$;

$\varphi = \pi(x + a)/2a$;

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$\psi = \pi(x-a)/2a$;
$2a$ is the length of primary mixture zone;
K is the effective turbulent diffusivity, m^2/sec;
t is the time counted from start of pumping, sec.

RELATION BETWEEN THERMAL AND ELECTRICAL CONDUCTIVITY OF GRAPHITE

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UDC 536.63.546.26-162

It is well known that the thermal conductivity and the electrical conductivity of graphite do not follow the Wiedemann–Frantz law. Heat transfer in graphite is effected by phonons, while electrical conduction is due to the flow of electrons and holes. Nevertheless, several authors have noted that at room temperature the product of thermal conductivity and electrical resistivity remains reasonably constant. No attempt was made so far to determine how these properties are related at high temperatures.

The article presents the results of thermal conductivity λ and electrical resistivity ρ measurements as well as calculated values of their product $\lambda \cdot \rho$ within the 80-2500°K range of temperature for synthetic graphite with a density ranging from 1.0 to 2.26 g/cm^3.

It has been established here that at low temperatures the values of $\lambda \cdot \rho$ for various grades of graphite vary widely. At room temperature the values of $\lambda \cdot \rho$ for the tested grades are closer together. Finally, at $T > 1500$°K the value of $\lambda \cdot \rho$ is 0.34-0.38 V^2/deg and independent of the temperature for all tested grades except those of the lowest density (1.0 g/cm^3) and the highest density (2.26 g/cm^3).

MEASURING THE DISPERSION AND THE VELOCITIES OF DROPLETS IN THE THROAT OF A VENTURI BY THE LIGHT-SCATTERING METHOD

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UDC 621.928.97

The mass distribution of droplets with respect to size was analyzed simultaneously at the center of the Venturi throat (section I) and behind the diffuser (section II), for various modes of water spraying after discharge from the center nozzle in the converging channel. The test conditions were such as to preclude either coagulation or fragmentation of droplets between the two illuminated sections. Functions $g_1(r)$ and $g_2(r)$ for one spray mode ($v_1 = 120$ m/sec, $m = 0.02$ kg/m^3, $L = 35$ mm) are shown in Fig. 1.

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†Lenin Polytechnic Institute, Kharkov. Original article submitted July 16, 1971; abstract submitted November 15, 1971.

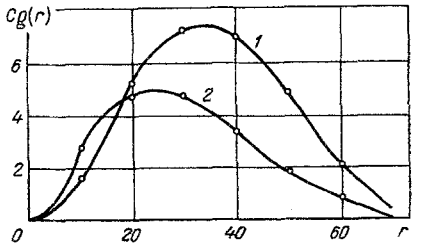

Fig. 1. Mass distribution of droplets with respect to size: 1) in the Venturi throat; 2) at the diffuser exit, for L = 35 mm and v_1 = 120 m/sec.

A higher degree of dispersion in the segment between sections I and II is explained by the gas carrying more effectively small droplets than large droplets through the Venturi throat, while the velocity of all droplets is almost the same and equal to the gas velocity in the diffuser.

From the continuity equation

$$m_1 g_1(r)\, dr v_1(r)\, S_1 = m_2 g_2(r)\, dr v_2(r)\, S_2, \tag{1}$$

and considering that $v_1(r) \to v_1$ at $r \to 0$ and $v_2(r) \approx v_2$, we have

$$v_1(r) = v_1 \frac{g_1(r)}{g_2(r)} \lim_{r \to 0} \frac{g_2(r)}{g_1(r)}. \tag{2}$$

The calculation of $v_1(r)$ by formula (2) is easy because, within the range of r = 2-10 μ, measurements of function g(r) by the light-scattering method are still very accurate and the ratio $v_1(r)/v_1$ approaches unity very slowly.

NOTATION

r	is the radius of droplet;
g(r)	is the mass distribution of droplets with respect to size;
v	is the velocity of gas at a given section;
v(r)	is the velocity of droplet with radius r;
L	is the distance between discharge orifices in nozzle and entrance to throat;
m	is the spray intensity;
mg(r)dr	is the mass of droplets of radii within the r, r + dr range, per unit of illuminated volume;
S	is the section area of the Venturi.

Subscripts

1	refers to section I;
2	refers to section II.

EFFECT OF STRUCTURAL POROSITY ON THE PROPAGATION OF ULTRASONIC WAVES THROUGH CAPILLARY-POROUS MODEL BODIES

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UDC 534.18

The physicomechanical properties of capillary-porous bodies are quite accurately described by a model-system of spherical elastic particles. The behavior of an elementary capillary-porous cell is determined by the elastic forces developing at the contact points between individual elements as well as by the friction forces appearing as a result of the relative displacement of particles.

Engineering Institute of the Light Industry, Kiev. Original article submitted May 20, 1971; abstract submitted November 11, 1971.

By analyzing the propagation of an elastic wave along a linear chain of capillary-porous cells, it is possible to show that, in the case of a negligible attenuation, the velocity of this wave does not depend on the size of particles. A measurement of the propagation velocity of ultrasonic waves in fractionalized quartz sand has shown an insignificant (24%) increase in the velocity of a 70 kHz wave over a significant range (factor 7.6) of particle sizes.

A linear chain of particles behaves like a mechanical low-pass filter which does not transmit elastic waves above the cutoff frequency. The attenuation of elastic waves at frequencies above cutoff is determined not by internal friction but by structural properties of the capillary-porous body. It appears that the cutoff frequency of the model capillary-porous body here is proportional to the wave velocity and inversely proportional to the particle size. According to calculations, the cutoff frequency of fractionalized quartz sand lies within the lower ultrasonic range.

CHOICE OF THE TIME INTERVAL IN SOLVING BOUNDARY-VALUE PROBLEMS BY THE LIEBMANN METHOD

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UDC 536.212

A wide range of scientific-technical problems can be reduced to solving equations of the following kind:

$$\sum_{i=1}^{3} \frac{\partial}{\partial x_i}\left(\mu \frac{\partial u}{\partial x_i}\right) = \omega \frac{\partial u}{\partial \tau} \tag{1}$$

with boundary conditions

$$\delta (u_m - u_s) = -\mu \frac{\partial u}{\partial n} \tag{2}$$

and initial conditions

$$u = f(x_1, x_2, x_3, \tau_{ini}) \tag{3}$$

The error of the solution to Eq. (1) obtained by the Liebmann method depends, with all other factors the same, on the choice of space interval Δx and time interval $\Delta\tau$. With an already selected space internal Δx, the accuracy of the solution will be affected principally by the choice of time interval $\Delta\tau$. A shorter $\Delta\tau$ will result in a better accuracy, but the computations become more laborious. Consequently, it is necessary to derive to devise a procedure for selecting the time interval $\Delta\tau$ to ensure the required accuracy at a minimum computation effort.

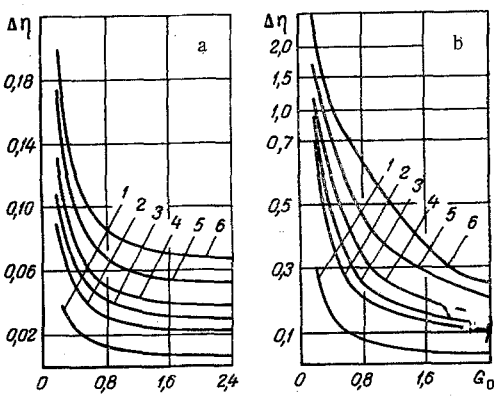

Fig. 1

In practical simulations the time interval $\Delta\tau$ is usually taken equal to 0.01-0.1% of the transient period in a process. The interval is then lengthened during computations depending on the time-variation of the potential and on the basis of past experience. The accuracy is evaluated by a comparison of several solutions obtained with different time intervals $\Delta\tau$.

An analysis of function $u = f(\tau)$ at the endpoints, for various values of the force-to-capacity ratio, has yielded

Institute of Maritime Engineers, Odessa. Original article submitted April 29, 1971; abstract submitted November 15, 1971.

criteria for selecting the time interval for a given accuracy requirement. This ratio is characterized by a quantity G_0 according to the definition

$$G_0 = \frac{\delta\tau_a}{\Delta x\omega} \cdot \frac{\mu}{\mu + \delta\Delta x}, \tag{4}$$

when the potential of the medium $u_m = K\tau$ for $\tau \in (0, \tau_a)$ and $u_m = K\tau_a$ for $\tau \in (\tau_a, \tau)$, with the field described by Eqs. (1)-(3). The curves of $\Delta\eta$ versus G_0 in Fig. 1 are useful for selecting the time interval as a function of G_0 and of the relative error $\Delta\theta/\theta$. Here $\eta = \tau/\tau_a$ is the dimensionless time and $\theta = u/k\tau_a$ is the dimensionless potential. Curves 1-6 have been plotted for a relative error $\Delta\theta/\theta = 0.2$, 0.4, 0.5, 0.6, 0.8, and 1.0% respectively. The graph in Fig. 1a is to be used for $\eta \in (0, (0.8 + G_0)/G_0)$, the graph in Fig. 1b is to be used for $\eta \in ((0.8 + G_0)/G_0, (5 + 1.6G_0)/G_0)$.

NOTATION

u	is the potential;
δ, μ, ω	are the coefficients;
x_i	is the space coordinates in (1), (3);
τ	is the time;
K	is the rate of change of potential.

Subscripts

s	refers to surface;
m	refers to medium.

AN EFFICIENT NUMERICAL INTEGRATION SCHEME FOR THE TWO-DIMENSIONAL TRANSIENT HEAT-CONDUCTION PROBLEM CONCERNING A VARIABLE-PROFILE DISK OF A GAS TURBINE

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UDC 621.438-254:536.2.001.2

The problem is formulated for a disk with a center hole. The original equation is

$$\frac{\partial t}{\partial \tau} = a\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial z^2}\right), \quad \tau > 0. \tag{1}$$

The boundary conditions are

$$\text{at } r = r_2 \quad \frac{\partial t}{\partial r} = \frac{\alpha(r_2)}{\lambda}[t - t_B(r_2)]; \tag{2}$$

$$\text{at } r = r_1 \quad \frac{\partial t}{\partial r} = \varphi_1 t + \varphi_2 t_{\text{л}}(\tau) + \varphi_3; \tag{3}$$

$$\text{at } z = f_1(r) \quad \frac{\partial t}{\partial n} = \frac{\alpha_1(r)}{\lambda}[t - t_{B_1}(r)]; \tag{4}$$

$$\text{at } z = f_2(r) \quad \frac{\partial t}{\partial n} = \frac{\alpha_2(r)}{\lambda}[t - t_{B_2}(r)] \tag{5}$$

(n denotes the normal to the lateral disk surface).

Institute of Aviation Engineering, Moscow. Original article submitted June 14, 1971; abstract submitted November 25, 1971.

For a numerical integration the curved domain is replaced by a Cartesian domain with the aid of the fractional-linear transformation

$$\xi = r \text{ and } \eta = \delta_1 \frac{z - f_1(r)}{f_2(r) - f_1(r)}. \tag{6}$$

Accordingly, Eq. (1) and the boundary conditions become

$$\frac{\partial t}{\partial \tau} = a\left[\frac{\partial B}{\partial r} \cdot \frac{\partial t}{\partial \eta} + 2B\frac{\partial^2 t}{\partial \xi \partial \eta} + \frac{\partial^2 t}{\partial \xi^2} + B^2\frac{\partial^2 t}{\partial \eta^2} + \frac{1}{\xi}\left(\frac{\partial t}{\partial \xi} + B\frac{\partial t}{\partial \eta}\right) + D^2\frac{\partial^2 t}{\partial \eta^2}\right] \tag{7}$$

$$(B = \partial\eta/\partial r;\ D = \partial\eta/\partial r);$$

$$\text{at } \xi = r_2 \quad \frac{\partial t}{\partial \xi} + B\frac{\partial t}{\partial \eta} = \frac{\alpha(r_2)}{\lambda}[t - t_{\text{в}}(r_2)]; \tag{8}$$

$$\text{at } \xi = r_1 \quad \frac{\partial t}{\partial \xi} + B\frac{\partial t}{\partial \eta} = \varphi_1 t + \varphi_2 t_{\text{л}}(\tau) + \varphi_3; \tag{9}$$

$$\text{at } \eta = 0 \quad D\frac{\partial t}{\partial \eta} = \frac{\alpha_1(r)}{\lambda}[t - t_{\text{в}_1}(r)]\cos\widehat{(n, z)}; \tag{10}$$

$$\text{at } \eta = \delta_1 \quad D\frac{\partial t}{\partial \eta} = -\frac{\alpha_2(r)}{\lambda}[t - t_{\text{в}_2}(r)]\cos\widehat{(n, z)}. \tag{11}$$

The solution is sought at the nodes of a uniform grid resulting from the intersection of straight lines $\xi_i = r_2 - ih_\xi$ ($i = 0, 1, 2, \ldots, n$) and $\eta_j = jh_\eta$ ($j = 0, 1, 2, \ldots, m$) for $\tau_k = kh_\tau$ ($k = 0, 1, 2, \ldots, s$).

The lengthwise–crosswise integration is performed in the following manner. The interval of integration with respect to τ is divided into two steps. Over the first half $0.5h_\tau$ Eq. (7) is integrated with respect to columns (along the ξ-axis). The difference representation is used here for the derivatives with respect ξ in layer $k + 1/2$ and for the derivatives with respect to η in layer k. Over the remaining second half $0.5h_\tau$ the integration is performed with respect to rows (along the η-axis). Here the difference representation is used for the derivatives with respect to ξ in layer $k + 1/2$ and for the derivatives with respect to η in layer $k + 1$.

The systems of equations

$$a_i\theta_{i-1}^{k+1/2} - b_i\theta_i^{k+1/2} + c_i\theta_{i+1}^{k+1/2} = F_i^k \text{ for columns} \tag{12}$$

$$a_j\theta_{j-1}^{k+1} - b_j\theta_j^{k+1} + c_j\theta_{j+1}^{k+1} = -F_j^{k+1/2} \text{ for rows} \tag{13}$$

are solved by the elimination method. Here a_i, b_i, c_i, a_j, b_j, and c_j are independent of τ. Functions F_i^k and $F_j^{k+1/2}$ contain derivatives with respect to ξ and η in the difference representation.

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SOLVING CERTAIN PROBLEMS IN THERMOPHYSICS ON AN ANALOG COMPUTER

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UDC 537.212.001

The solution of great many application problems in thermophysics by simulation is fraught with serious difficulties. The reason is that the use of existing models is essentially limited to solving direct

Lenin Polytechnic Institute, Kharkov. Original article submitted June 15, 1971; abstract submitted November 17, 1971.

problems in field theory. At the same time, most practical problems are formulated more in terms of reverse, inverse, or inductive problems. Consequently, often only an insignificant part of the solution to an application problem can be obtained by simulation and the main part of the problem must be solved by other means and methods of computer engineering.

This article deals with the design and principles of analog computer systems (ACS) for a direct solution of certain application problems in thermophysics. The solution process is in this case a continuous one and does not require auxiliary computations.

A typical functional schematic diagram of an analog computer system is shown here containing an RC-network for simulating the test object and a device for setting the boundary conditions (DSBC). The latter component represents a known model for solving direct problems in field theory. An essential feature of this analog computer system is a control unit (CU) which, together with the DSBC sets the boundary conditions according to the values of the derivatives of field characteristics obtained on the computing unit. In other words, the results of solving a direct problem in field theory operationally on the RC-network are used for the electronic simulation of such boundary conditions which will ensure that the analyzed process continues in the desired direction.

For illustration, the authors describe an analog computer system designed on the basis of USM-1 computer elements and show the results of solving on it one practical thermophysics problem. The particular problem concerns a heat carrier flowing from inlet to outlet between two coaxial cylinders, and the temperature–time characteristic is sought which will ensure a transition of this system from initial to operating state in minimum time. The operating state of this system is defined by the maximum steady-state temperature of the heat carrier. The control process must satisfy the requirement that the relative displacement between the free ends of the cylinders do not exceed the allowable limit, this displacement being equal to the difference between their thermal elongations. Such a problem is often encountered in applications as, for example, determining the startup conditions in a steam turbine.

The optimum temperature–time curve for the heat carrier thus found consists of two distinct ranges. Within one range the temperature reaches its maximum, the other range corresponds to a displacement of the controlled system to the boundary value of the measured parameter. The proposed structure of such an analog computer system yields a basis for solving more complicated application problems. The model structure is here determined by the problem formulation and should reflect the nature of the constraints as well as the requirement imposed on the simulated process.

CYLINDRICAL AND SPHERICAL THERMOELASTIC WAVES IN THE SHORT-TIME DOMAIN WITH A FINITE VELOCITY OF HEAT PROPAGATION

I. K. Naval and P. F. Sabodash

UDC 539.377:536.49

The authors analyze the excitation of one-dimensional cylindrical and spherical thermoelastic waves in elastic and isotropic cylindrical (spherical) layers of finite thickness as a result of temperature jumps at the inside surface of such layers. The initial temperature of a layer is equal to the temperature of the ambient medium. The thermophysical properties are assumed constant.

On the assumption that the temperature field follows the hyperbolic equation of heat conduction, the problem reduces to integrating a system of uncoupled differential equations (in dimensionless variables)

$$\frac{\partial^2 v}{\partial \xi^2} + \frac{\nu}{\xi} \cdot \frac{\partial v}{\partial \xi} - \frac{\nu v}{\xi^2} = \frac{\partial^2 v}{\partial \tau^2} + \omega \frac{\partial \theta}{\partial \xi}, \quad (1)$$

Original article submitted June 14, 1971; abstract submitted November 25, 1971.

$$\frac{\partial^2\theta}{\partial\xi^2}+\frac{\nu}{\xi}\cdot\frac{\partial\theta}{\partial\xi}=M^2\frac{\partial^2\theta}{\partial\tau^2}+\frac{\partial\theta}{\partial\tau}, \tag{2}$$

inside the layer with the boundary conditions

$$\frac{\partial v}{\partial\xi}+\nu\gamma\frac{v}{\xi}=\omega_1 \text{ at } \xi=\xi_0;\quad \frac{\partial v}{\partial\xi}+\nu\gamma\frac{v}{\xi}=\omega_2 \text{ at } \xi=\xi_1;$$

$$\theta(\xi,\tau)=H(\tau) \text{ at } \xi=\xi_0;\ \theta(\xi,\tau)=0 \text{ at } \xi=\xi_1$$

and zero initial conditions

$$v=\frac{\partial v}{\partial\tau}=\theta=\frac{\partial\theta}{\partial\tau}=0 \text{ at } \tau\leqslant 0,$$

where

$$\theta=\frac{T-T_0}{T_c-T_0},\quad \xi=\frac{cr}{a},\quad \tau=\frac{c^2t}{a},\quad v=\frac{u}{r_0},\quad M=\frac{c}{c_q},$$

$$\omega_1=\frac{(3\lambda+2\mu)\alpha T_c}{(\lambda+2\mu)\xi_0},\quad \omega_2=\frac{(3\lambda+2\mu)\alpha T_0}{(\lambda+2\mu)\xi_0},\quad \gamma=\frac{\lambda}{\lambda+2\mu},\quad \omega=\omega_1-\omega_2;$$

$\nu = 1$ for a cylindrical wave and $\nu = 2$ for a spherical wave.

System (1)-(2) is solved with the aid of the unilateral Laplace transformation with respect to time.

An exact temperature distribution has been obtained for the inside of a sphere. In the case of a cylinder, an exact transformation is difficult because of the unwieldy expressions for the temperature and the displacement in terms of the Laplace operator.

An asymptotic transformation is shown here for large values of the Laplace operator, corresponding to short time periods, and the solution becomes valid in the region of incident and multiply reflected (at both boundary surfaces) thermoelastic wave fronts.

NOTATION

$T(r, t)$	is the temperature;
$u(r, t)$	is the radial component of the elastic displacement vector;
a	is the thermal diffusivity;
c_q	is the velocity of heat propagation;
r	is the radial coordinate;
t	is the time;
λ, μ	are the Lamé constants;
c	is the velocity of elastic waves;
α	is the thermal expansivity;
$H(\tau)$	is the Heaviside function.

A THEOREM ON THE FINITE INTEGRAL HANKEL TRANSFORMATION AND ITS APPLICATION

O. N. Kharin and V. S. Blinov

UDC 519.47

The authors consider the finite integral Hankel transformation of the first order, for which the following theorem is proved:

Gubkin Institute of the Petrochemical and Natural Gas Industry, Moscow. Original article submitted July 2, 1970; abstract submitted November 9, 1971.

THEOREM. Let functions f(r) and g(r) satisfy the Dirichlet conditions on a close interval [0, 1]

$$g(r) = \int_r^1 \frac{dr}{r} \int_0^r rf(r)\, dr. \tag{1}$$

Then

1° if $f(r) \doteq F(a_n)$,

$$g(r) \doteq \frac{F(a_n)}{a_n^2}; \tag{2}$$

2° if $f(r) \doteq \begin{cases} F(b_0), & n = 0 \\ F(b_n), & n = 1, 2, \ldots, n \end{cases}$

is known, then

$$g(r) \doteq \begin{cases} \frac{1}{4} F(b_0) - \frac{1}{4} \int_0^1 r^3 f(r)\, dr, & n = 0, \\ \frac{F(b_n)}{b_n^2} - \frac{J_0(b_n)}{b_n^2} F(b_0), & n = 1, 2, \ldots; \end{cases} \tag{3}$$

3° if $f(r) \doteq F(c_n)$,

$$g_1(r) \doteq \frac{F(c_n)}{c_n^2} - \frac{J_1(c_n)}{hc_n} \int_0^1 rf(r)\, dr, \tag{4}$$

with a_n, b_n, and c_n denoting the roots of characteristic equations corresponding to the first, the second, and the third kind respectively.

COROLLARY 1. If the sum $f_j(r)$ of the series

$$f_j(r) = 2 \sum_{n=1}^{\infty} \frac{J_0(a_n r)}{J_1^2(a_n)} \cdot \frac{F(a_n)}{a_n^{2j}} \quad (j = 0, 1, 2, \ldots) \tag{5}$$

is known, then the sum $f_{j+1}(r)$ of series (5) can be calculated by the formula

$$f_{j+1}(r) = \int_r^1 \frac{dr}{r} \int_0^r rf_j(r)\, dr. \tag{6}$$

COROLLARY 2. If the sum $f_j(r)$ of the series

$$f_j(r) = 2 \sum_{n=1}^{\infty} \frac{J_0(rb_n)}{J_0^2(b_n)} \cdot \frac{F(b_n)}{b_n^{2j}} \quad (j = 0, 1, 2, \ldots) \tag{7}$$

is known, then

$$f_{j+1}(r) = \int_r^1 \frac{dr}{r} \int_0^r rf_j(r)\, dr + \frac{1}{2} \int_0^1 r^3 f_j(r)\, dr. \tag{8}$$

COROLLARY 3. If the sum $f_j(r)$ of the series

$$f_j(r) = 2 \sum_{n=1}^{\infty} \frac{J_0(c_n r)}{J_0^2(c_n) + J_1^2(c_n)} \cdot \frac{F(c_n)}{c_n^{2j}} \quad (j = 0, 1, 2, \ldots) \tag{9}$$

is known, then

$$f_{j+1}(r) = \int_r^1 \frac{dr}{r} \int_0^r rf_j(r)\, dr + \frac{1}{h} \int_0^1 rf_j(r)\, dr. \tag{10}$$

COROLLARY 4. If the sum $f_j(r)$ of one of the series (5), (7), (9) is defined by different functions on different intervals of r, then also

$$f_{j+1}(r) = \begin{cases} f_{j+1}^{(e)}(r) & (R \leqslant r \leqslant 1), \\ f_{j+1}^{(i)}(r) & (0 \leqslant r \leqslant R), \end{cases} \tag{11}$$

where

$$f_{j+1}^{(e)}(r) = \int_r^1 \frac{dr}{r} \int_R^r r f_j^{(e)}(r)\, dr - \ln r \int_0^R r f_j^{(i)}(r)\, dr + \psi(r), \tag{12}$$

$$f_{j+1}^{(i)}(r) = \int_r^R \frac{dr}{r} \int_0^r r f_j^{(i)}(r)\, dr + f_{j+1}^{(e)}(R). \tag{13}$$

Here the value of function $\psi(r)$ depends on the kind of the summed series

$$\psi(r) = \begin{cases} 0, \\ \dfrac{1}{2} \displaystyle\int_0^1 r^3 f_j(r)\, dr, \\ \dfrac{1}{h} \displaystyle\int_0^1 r f_j(r)\, dr. \end{cases} \tag{14}$$

The first value of $\psi(r)$ corresponds to a summation of a series like (5), the second value to a summation of a series like (7), and the third value to a summation of a series like (9).

The application of these results is illustrated in the problem of heating a thermally insulated cylinder from variable-power sources uniformly distributed within a certain region of the cylinder.